Package ‘SSReliabilityClaytonMWD’

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Title Stress-Strength Reliability Model with MWD Marginals via Clayton Copula

Version 1.0.2

Description Implements stress-strength reliability models under a dependent framework, where both stress and strength variables follow modified Weibull distributions and their dependence is modeled using a Clayton copula (Kizilaslan (2026) <doi:10.48550/arXiv.2604.12130>).
The package provides several estimation procedures for model parameters and the stress-strength reliability R, including two-step maximum likelihood estimation (MLE), least squares estimation (LSE), weighted least squares estimation (WLSE), and maximum product of spacings (MPS). It also provides interval estimation using asymptotic confidence intervals based on MLE and bootstrap confidence intervals for all methods.
In addition, functions are included for parameter estimation of the modified Weibull distribution (Lai et al. (2003) <doi:10.1109/TR.2002.805788>) and the two-parameter Weibull distribution, along with utilities to compute their probability density function, cumulative distribution function, quantile function, and to generate random samples.

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URL https://github.com/fatihki/SSReliabilityClaytonMWD, https://fatihki.github.io/SSReliabilityClaytonMWD/

BugReports https://github.com/fatihki/SSReliabilityClaytonMWD/issues

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Contents

`AllDams` *All Istanbul Dams Data*

Description

Daily data for 10 dams in Istanbul, Türkiye. The dataset consists of daily occupancy rates of Istanbul's dams, retrieved in March 2026 from https://data.ibb.gov.tr/en. The data span the period from late October 2000 to mid-February 2024.

Usage

```
AllDams
```

Format

A data frame with 8520 rows and 13 variables.

Source

Istanbul Metropolitan Municipality Open Data Portal https://data.ibb.gov.tr/en. Licensed under CC BY 4.0.

`Clayton_Copula` *Two-dimensional Clayton Copula*

Description

Computes the joint cumulative distribution function (CDF) and probability density function (PDF) of the two-dimensional Clayton copula.

Usage

```
Clayton_Copula(u, v, theta)

Clayton_Copula_pdf(u, v, theta)
```

Arguments

`u`	Numeric vector of values in $[0, 1]$. First marginal (uniform).
`v`	Numeric vector of values in $[0, 1]$. Second marginal (uniform).
`theta`	Positive numeric scalar. Dependence parameter $\theta > 0$.

Details

The joint distribution function of the two-dimensional Clayton copula is

$$C(u, v; \theta) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-1/\theta},$$

where $\theta > 0$.

The corresponding joint density is given by

$$c(u, v; \theta) = (\theta + 1)u^{-(\theta+1)}v^{-(\theta+1)} \left(u^{-\theta} + v^{-\theta} - 1\right)^{-(1/\theta+2)}.$$

Value

- `Clayton_Copula`: Numeric vector of CDF values.
- `Clayton_Copula_pdf`: Numeric vector of PDF values.

References

Nelsen, R. B. (2006). *An Introduction to Copulas*. Springer.

Examples

```
u <- c(0.2, 0.5, 0.8)
v <- c(0.3, 0.6, 0.9)

Clayton_Copula(u, v, theta = 2)
Clayton_Copula_pdf(u, v, theta = 2)
```

---

fit.SSR.ClaytonMWD *Estimation of SSR Model Parameters with MWD Marginals via Clayton Copula*

Description

Fits a dependent stress–strength reliability (SSR) model in which both strength and stress follow the Modified Weibull Distribution (MWD), and dependence is modeled using a Clayton copula.

The function estimates marginal parameters (a_1, b_1, λ_1) for strength X, (a_2, b_2, λ_2) for stress $(Y$, and the copula dependence parameter θ.

Estimation is performed using Maximum Likelihood Estimation (MLE), Least Squares Estimation (LSE), Weighted Least Squares Estimation (WLSE), and Maximum Product of Spacings (MPS).

Usage

```
fit.SSR.ClaytonMWD(
  data,
  ACI = FALSE,
  bootstrap = FALSE,
  B = NULL,
  seed = NULL,
  one.step = TRUE,
  alpha = 0.05,
  verbose = FALSE
)
```

Arguments

data	A list containing two numeric vectors: X (strength) and Y (stress).
ACI	Logical. If TRUE, asymptotic 95% confidence intervals based on MLE are computed.
bootstrap	Logical. If TRUE, parametric bootstrap confidence intervals are computed.
B	Integer. Number of bootstrap replications.
seed	Integer. Random seed for reproducibility.
one.step	Logical. If TRUE,one-step LSE and WLSE estimators are used for θ.
alpha	Numeric. Significance level for confidence intervals (e.g., 0.05 for a 95% confidence interval).
verbose	Logical; if TRUE, progress and intermediate results from the optimization procedure are printed. Default is FALSE.

Details

Fit SSR Model with Modified Weibull Marginals via Clayton Copula

Returns point estimates and interval estimates of model parameters using MLE, LSE, WLSE, and MPS methods.

Further theoretical details are available in Kizilaslan (2026).

Value

A list containing:

`all.results`	Point estimates of all model parameters.
`theta.Ktau`	Kendall's tau estimate corresponding to θ.
`seed`	Random seed used in the analysis.
`data`	Input dataset used in the analysis.
`ACI.parameters`	If `ACI = TRUE`, asymptotic 95% confidence intervals for model parameters.
`boot.mle`	If `bootstrap = TRUE`, bootstrap confidence intervals based on MLE.
`boot.lse`	If `bootstrap = TRUE`, bootstrap confidence intervals based on LSE.
`boot.wlse`	If `bootstrap = TRUE`, bootstrap confidence intervals based on WLSE.
`boot.mps`	If `bootstrap = TRUE`, bootstrap confidence intervals based on MPS.
`boot.samples`	A list containing bootstrap samples for all parameters across all methods.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv:2604.12130

Examples

```
data <- list(X = TerkosDam, Y = OmerliDam)
fit.SSR <- fit.SSR.ClaytonMWD(data,  ACI = TRUE, bootstrap = TRUE, B = 5,
                              seed = 2026, one.step = TRUE, alpha = 0.05)
print(fit.SSR)
```

`fitClayton` *Estimation of the Clayton Copula Dependence Parameter*

Description

Estimates the dependence parameter θ of the Clayton copula based on observed data from a stress–strength model.

Usage

```
fitClayton(
  x,
  y,
  est.method,
  opt.method,
  start,
  estimates,
  lower = NULL,
  upper = NULL,
  verbose = FALSE,
  ...
)
```

Arguments

`x`	Numeric vector. Observations of the strength variable X.
`y`	Numeric vector. Observations of the stress variable Y.
`est.method`	Character string specifying the estimation method used. Options include `"MLE"`, `"LSE"`, `"WLSE"`, and `"MPS"`.
`opt.method`	Character string specifying the optimization method used in `optim`. Common options include `"Nelder-Mead"`, `"BFGS"`, `"CG"`, `"L-BFGS-B"`, `"SANN"`, and `"Brent"`.
`start`	Numeric scalar. Initial value for θ.
`estimates`	A named list of estimated marginal parameters: (a_1, b_1, λ_1) for strength and (a_2, b_2, λ_2) for stress.
`lower`	Numeric vector. Lower bounds for parameters in constrained optimization. Only used if supported by `opt.method`.
`upper`	Numeric vector. Upper bounds for parameters in constrained optimization. Only used if supported by `opt.method`.
`verbose`	Logical; if `TRUE`, progress and intermediate results from the optimization procedure are printed. Default is `FALSE`.
`...`	Additional arguments passed to `optim`.

Details

Estimate the Clayton Copula Parameter

The Clayton copula is defined as

$$C(u, v; \theta) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-1/\theta},$$

where $\theta > 0$.

The parameter is estimated using the following methods:

- **Maximum Likelihood Estimation (MLE):** Maximizes the joint log-likelihood under the assumed model.

- **Least Squares Estimation (LSE):** Minimizes squared differences between empirical and theoretical CDFs. The empirical CDF uses Benard's approximation: $F(x_{(i)}) = (i - 0.3)/(n + 0.4)$, for $i = 1, \ldots, n$.
- **Weighted Least Squares Estimation (WLSE):** Uses weights $w_i = \frac{(n+1)^2(n+2)}{i(n-i+1)}$, for $i = 1, \ldots, n$.
- **Maximum Product of Spacings (MPS):** Maximizes the product of spacings of the fitted distribution function, providing a robust alternative to MLE.

Further theoretical details are provided in Kizilaslan (2026).

Value

A list containing:

`estimate`	Estimate of the Clayton copula parameter, θ.
`opt.fit`	Full optimization result.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv preprint. Available at arXiv:2604.12130.

`fitMWD` *Estimation of Parameters for the Modified Weibull Distribution*

Description

Estimates the parameters of the Modified Weibull Distribution (MWD) using classical estimation methods.

Usage

```
fitMWD(
  data,
  est.method,
  opt.method,
  starts,
  lower = NULL,
  upper = NULL,
  verbose = FALSE,
  ...
)
```

Arguments

`data`	Numeric vector of observations.
`est.method`	Character string specifying the estimation method. Options include `"MLE"`, `"LSE"`, `"WLSE"`, and `"MPS"`.
`opt.method`	Character string specifying the optimization method used in `optim`, such as `"Nelder-Mead"`, `"BFGS"`, `"CG"`, `"L-BFGS-B"`, `"SANN"`, or `"Brent"`.
`starts`	Numeric vector of initial values for the parameters
`lower`	Numeric vector of lower bounds for parameters in constrained optimization. Ignored if `NULL`.
`upper`	Numeric vector of upper bounds for parameters in constrained optimization.
`verbose`	Logical. If `TRUE`, prints optimization progress.
`...`	Additional arguments passed to `optim`.

Details

Fit the Modified Weibull Distribution (MWD)

The Modified Weibull Distribution (Lai et al., 2003) has cumulative distribution function (CDF) and probability density function (PDF):

$$F(x) = 1 - \exp\left(-ax^b \exp(\lambda x)\right),$$
$$f(x) = a(b + \lambda x)x^{b-1} \exp(\lambda x) \exp\left(-ax^b \exp(\lambda x)\right),$$

where $x > 0$, $a > 0$ is a scale parameter, $b \geq 0$ is a shape parameter, and $\lambda \geq 0$ is a flexibility parameter controlling the growth rate of the hazard function.

The parameters are estimated using the following methods:

- **Maximum Likelihood Estimation (MLE):** Maximizes the log-likelihood under the MWD model.
- **Least Squares Estimation (LSE):** Minimizes squared differences between empirical and theoretical CDFs. The empirical CDF uses Benard's approximation: $F(x_{(i)}) = (i - 0.3)/(n + 0.4)$, for $i = 1, \ldots, n$.
- **Weighted Least Squares Estimation (WLSE):** A modification of LSE that assigns weights to the squared differences. Uses weights $w_i = \frac{(n+1)^2(n+2)}{i(n-i+1)}$, for $i = 1, \ldots, n$.
- **Maximum Product of Spacings (MPS):** Maximizes the product of spacings of the fitted CDF.

Further details are provided in Kizilaslan (2026).

Value

A list containing:

`estimates`	Named numeric vector of estimated parameters (a, b, λ).
`measures`	Numeric vector of model selection criteria (log-likelihood, AIC, BIC).
`initials`	Initial values used in the optimization.
`opt.fit`	Full output from `optim`.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv preprint. Available at arXiv:2604.12130.

Lai, C. D., Xie, M., and Murthy, D. N. P. (2003). A modified Weibull distribution. *IEEE Transactions on Reliability*, **52**(1), 33–37.

Examples

```
# generate data from MWD(a, b, lambda)
n <- 100
a <- 0.75; b <- 1.25; lambda <- 0.60
set.seed(123)
dat <- rMweibull(n, a, b, lambda)
init <- runif(3)

# Fit MWD to dat.
fit.mle <- fitMWD(data = dat, est.method = "mle", opt.method = "L-BFGS-B", starts = init,
                  lower = c(1e-05,1e-05,1e-05), upper = c(Inf,Inf,Inf), hessian = FALSE )
fit.mle$estimates

fit.lse <- fitMWD(data = dat, est.method = "lse", opt.method = "L-BFGS-B", starts = init,
                  lower = c(1e-05,1e-05,1e-05), upper = c(Inf,Inf,Inf), hessian = FALSE )
fit.lse$estimates

fit.wlse <- fitMWD(data = dat, est.method = "wlse", opt.method = "L-BFGS-B", starts = init,
                  lower = c(1e-05,1e-05,1e-05), upper = c(Inf,Inf,Inf), hessian = FALSE )
fit.wlse$estimates

fit.mps <- fitMWD(data = dat, est.method = "mps", opt.method = "L-BFGS-B", starts = init,
                  lower = c(1e-05,1e-05,1e-05), upper = c(Inf,Inf,Inf), hessian = FALSE )
fit.mps$estimates
```

---

fitWD — *Estimating parameters of the two-parameter Weibull distribution*

Description

Estimates the parameters of the two-parameter Weibull distribution using classical methods.

Usage

```
fitWD(
  data,
  est.method,
  opt.method,
  starts,
  lower = NULL,
```

```
    upper = NULL,
    verbose = FALSE,
    ...
)
```

Arguments

`data`	Numeric vector of observations.
`est.method`	Character string specifying the estimation method. Options include `"MLE"`, `"LSE"`, `"WLSE"`, and `"MPS"`.
`opt.method`	Character string specifying the optimization method used in `optim`, such as `"Nelder-Mead"`, `"BFGS"`, `"CG"`, `"L-BFGS-B"`, `"SANN"`, or `"Brent"`.
`starts`	Numeric vector of initial values for the parameters
`lower`	Numeric vector of lower bounds for parameters in constrained optimization. Ignored if `NULL`.
`upper`	Numeric vector of upper bounds for parameters in constrained optimization.
`verbose`	Logical. If `TRUE`, prints optimization progress.
`...`	Additional arguments passed to `optim`.

Details

Classical estimations fo the parameters of the two-parameter Weibull distribution

The two-parameter Weibull Distribution has cumulative distribution function (CDF) and probability density function (PDF):

$$F(x) = 1 - \exp(-ax^b),$$

$$f(x) = abx^{b-1}\exp(-ax^b),$$

where $x > 0$, $a > 0$ is the scale parameter and $b > 0$ is the shape parameter.

The parameters are estimated using the following methods:

- **Maximum Likelihood Estimation (MLE):** Maximizes the log-likelihood under the MWD model.
- **Least Squares Estimation (LSE):** Minimizes squared differences between empirical and theoretical CDFs. The empirical CDF uses Benard's approximation: $F(x_{(i)}) = (i - 0.3)/(n + 0.4)$, for $i = 1, \ldots, n$.
- **Weighted Least Squares Estimation (WLSE):** A modification of LSE that assigns weights to the squared differences. Uses weights $w_i = \frac{(n+1)^2(n+2)}{i(n-i+1)}$, for $i = 1, \ldots, n$.
- **Maximum Product of Spacings (MPS):** Maximizes the product of spacings of the fitted CDF.

Value

A list containing:

`estimates`	Named numeric vector of estimated parameters (a, b).
`measures`	Numeric vector of model selection criteria (log-likelihood, AIC, BIC).
`initials`	Initial values used in the optimization.
`opt.fit`	Full output from `optim`.

Examples

```
# generate data from WD(a, b)
n <- 50
a <- 0.75; b <- 1.25; lambda <- 0 # reduces two-parameter Weibull distribution
set.seed(123)
X <- rMweibull(n, a, b, lambda)
init <- runif(2)

# fit model
fit.mle <- fitWD(data = X, est.method = "mle", opt.method = "L-BFGS-B", starts = init,
                 lower = c(1e-05,1e-05), upper = c(Inf,Inf), hessian = FALSE )
fit.mle$estimates

fit.lse <- fitWD(data = X, est.method = "lse", opt.method = "L-BFGS-B", starts = init,
                 lower = c(1e-05,1e-05), upper = c(Inf,Inf), hessian = FALSE )
fit.lse$estimates

fit.wlse <- fitWD(data = X, est.method = "wlse", opt.method = "L-BFGS-B", starts = init,
                  lower = c(1e-05,1e-05), upper = c(Inf,Inf), hessian = FALSE )
fit.wlse$estimates

fit.mps <- fitWD(data = X, est.method = "mps", opt.method = "L-BFGS-B", starts = init,
                 lower = c(1e-05,1e-05), upper = c(Inf,Inf), hessian = FALSE )
fit.mps$estimates
```

`LSE_clayton_onestep` *One-Step Least Squares Estimation of the Clayton Copula Parameter*

Description

Computes a one-step least squares estimator (LSE) of the Clayton copula dependence parameter θ. The estimator is obtained via a second-order Taylor expansion of the Clayton copula $C_\theta(u, v)$ around an initial value θ_0, typically the Kendall's tau-based moment estimate.

Usage

```
LSE_clayton_onestep(par, x, y, estimates)
```

Arguments

`par`	Numeric scalar. Initial estimate of θ, typically obtained from Kendall's tau.
`x`	Numeric vector. Observations of the strength variable X.
`y`	Numeric vector. Observations of the stress variable Y.
`estimates`	A named list of marginal parameter estimates: (a_1, b_1, λ_1) for strength and (a_2, b_2, λ_2) for stress.

Details

One-Step LSE Estimator for the Clayton Copula Parameter

The one-step estimator is constructed by substituting a second-order Taylor expansion of the Clayton copula $C_\theta(u, v)$ into the least squares estimating equation, evaluated at θ_0, and solving analytically for θ.This avoids iterative numerical optimisation and yields a closed-form estimation of θ.

Further theoretical details are provided in Kizilaslan (2026).

Value

Numeric scalar. One-step LSE estimate of the dependence parameter θ.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv:2604.12130

See Also

`WLSE_clayton_onestep` for the weighted LSE version, `theta_Ktau_estimate` for the Kendall's Tau-based estimate.

`ModifiedWeibull` *The Modified Weibull Distribution (MWD)*

Description

Density, distribution function, quantile function, random generation, and hazard function for the Modified Weibull distribution (MWD) introduced by Lai et al. (2003).

Usage

```
dMweibull(x, a, b, lambda, log = FALSE)

pMweibull(q, a, b, lambda, lower.tail = TRUE, log = FALSE)

qMweibull(p, a, b, lambda, lower.tail = TRUE)
```

```
rMweibull(n, a, b, lambda)

hMweibull(x, a, b, lambda, log = FALSE)
```

Arguments

`x`	Numeric vector of observations.
`a`	Positive scale parameter ($a > 0$).
`b`	Non-negative shape parameter ($b \geq 0$).
`lambda`	Non-negative parameter ($\lambda \geq 0$) controlling the growth rate of the hazard function.
`log`	Logical. If `TRUE`, returns log-density, log-distribution, or log-hazard values where applicable.
`q`	Numeric vector of quantiles.
`lower.tail`	Logical. If `FALSE`, returns $1 - F(x)$ and computes quantiles for $1 - p$. #' The Modified Weibull distribution with parameters a, b, and λ has cumulative distribution function (CDF), probability density function (PDF), and hazard function given by
`p`	Numeric vector of probabilities in $[0, 1]$.
`n`	Integer; number of observations to be generated.

Details

Modified Weibull Distribution

The Modified Weibull distribution with parameters a, b and λ has cumulative distribution function (CDF), probability density function (PDF), and hazard function given by

$$F(x) = 1 - \exp\left(-ax^b \exp(\lambda x)\right),$$
$$f(x) = a(b + \lambda x)x^{b-1} \exp(\lambda x) \exp\left(-ax^b \exp(\lambda x)\right),$$
$$h(x) = a(b + \lambda x)x^{b-1} \exp(\lambda x),$$

where $x > 0$, $a > 0$ is the scale parameter, $b \geq 0$ is a shape parameter, and $\lambda \geq 0$ is an acceleration or flexibility parameter that controls how quickly the hazard grows over time.

Special cases:

- If $\lambda = 0$, the MWD reduces to the Weibull distribution $F(x) = 1 - \exp(-ax^b)$.
- If $b = 0$, the MWD reduces to a type I extreme-value (log-gamma) distribution $F(x) = 1 - \exp(-a \exp(\lambda x))$.

Value

- `dMweibull`: Density values.
- `pMweibull`: Distribution function values.
- `qMweibull`: Quantiles.
- `rMweibull`: Random deviates.
- `hMweibull`: Hazard function values.

References

Lai, C. D., Xie, M., and Murthy, D. N. P. (2003). A modified Weibull distribution. *IEEE Transactions on Reliability*, **52**(1), 33–37.

Examples

```
n <- 25
a <- 0.75; b <- 1.25; lambda <- 0.60
set.seed(123)
x <- rlnorm(n)
ff <- dMweibull(x, a, b, lambda)
FF <- pMweibull(x, a, b, lambda)
qq <- qMweibull(runif(n), a, b, lambda)
dat <- rMweibull(n, a, b, lambda)
hf <- hMweibull(x, a, b, lambda )
```

OmerliDam *Omerli Dam Data*

Description

Omerli Dam is the largest dam supplying Istanbul, Türkiye, and is located on the Anatolian side. The dataset consists of daily occupancy rates of Istanbul's dams, retrieved in March 2026 from Istanbul Metropolitan Municipality datasets website https://data.ibb.gov.tr/en.

Usage

```
OmerliDam
```

Format

A numeric vector of length 95, representing monthly average occupancy rates.

Details

The data span the period from late October 2000 to mid-February 2024. Monthly average occupancy rates are computed based on the daily data for the period September-December of each year, resulting in a total of 95 observations.

Source

Istanbul Metropolitan Municipality Open Data Portal https://data.ibb.gov.tr/en. Licensed under CC BY 4.0.

parametric_bootstrap *Parametric Bootstrap Confidence Intervals*

Description

Computes parametric bootstrap confidence intervals for unknown model parameters and reliability R, based on maximum likelihood estimation (MLE), least squares estimation (LSE), weighted least squares estimation (WLSE), and maximum product of spacing estimation (MPS).

Usage

```
parametric_bootstrap(
  est.method,
  opt.method,
  boot.estimates,
  n,
  B = 1000,
  seed = NULL,
  one.step = TRUE,
  alpha = 0.05
)
```

Arguments

`est.method`	Character string specifying the estimation method used. Options include `"MLE"`, `"LSE"`, `"WLSE"`, and `"MPS"`.
`opt.method`	Character string specifying the optimization method used in `optim`. Common options include `"Nelder-Mead"`, `"BFGS"`, `"CG"`, `"L-BFGS-B"`, `"SANN"`, and `"Brent"`.
`boot.estimates`	A named list of initial parameter estimates. The elements (a_1, b_1, λ_1) correspond to the strength variable, (a_2, b_2, λ_2) correspond to the stress variable, and θ is the Clayton copula dependence parameter.
`n`	Integer. Sample size.
`B`	Integer. Number of bootstrap replications.
`seed`	Integer. Random seed for reproducibility.
`one.step`	Logical. If `TRUE`, one-step LSE and WLSE estimators are used for θ.
`alpha`	Numeric. Significance level for confidence intervals (e.g., `0.05` for a 95% confidence interval).

Details

This function implements a parametric bootstrap percentile method to construct confidence intervals for unknown parameters and reliability R under different estimation methods (MLE, LSE, WLSE, and MPS).

Further theoretical details are provided in Kizilaslan (2026).

Value

A list containing:

`parameters.quantiles`
: A numeric matrix with lower and upper $100(1-\alpha)\%$ bootstrap percentile confidence limits.

`boot.results`
: A matrix of bootstrap estimates for all parameters over B replications.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv:2604.12130

`print.SSRfit` *Print Method for SSR Fit Objects*

Description

Prints results of an object of class `SSRfit` produced by `fit.SSR.ClaytonMWD`. The output includes parameter estimates and confidence interval estimates obtained using MLE, LSE, WLSE, and MPS, for the marginal parameters, the Clayton copula parameter θ, and the reliability measure R.

Usage

```
## S3 method for class 'SSRfit'
print(x, ...)
```

Arguments

`x`
: An object of class `SSRfit` returned by `fit.SSR.ClaytonMWD`.

`...`
: Additional arguments passed to the print method. For example, `digits` controls the number of decimal places used in printed output.

Details

Print SSR Fit Results (Clayton Copula with MWD Marginals)

This method organizes and displays results in a structured format, separating point estimates and interval estimates for all model components.

Value

Invisibly returns the input object `x` of class `"SSRfit"`. The function is called for its side effects, namely printing formatted summaries of parameter estimates, dependence parameter estimates, and associated confidence intervals to the console.

Examples

```
data = list(X = TerkosDam, Y = OmerliDam)
fit.SSR = fit.SSR.ClaytonMWD(data, ACI = TRUE, bootstrap = TRUE, B = 5,
                             seed = 2026, one.step = TRUE, alpha = 0.05)
print(fit.SSR)
print(fit.SSR, 3)
```

`Reliability_Clayton_MWD`
Stress–Strength Reliability for MWD under Clayton Copula

Description

Computes the stress–strength reliability (SSR) $R = P(X > Y)$, where $X \sim \mathrm{MWD}(a_1, b_1, \lambda_1)$ (strength) and $Y \sim \mathrm{MWD}(a_2, b_2, \lambda_2)$ (stress), with dependence modeled using a Clayton copula.

Usage

```
Reliability_Clayton_MWD(a1, b1, lambda1, a2, b2, lambda2, theta)
```

Arguments

`a1, b1, lambda1`	Parameters of the strength variable X, with $a_1 > 0$, $b_1 \geq 0$, and $\lambda_1 \geq 0$.
`a2, b2, lambda2`	Parameters of the stress variable Y, with $a_2 > 0$, $b_2 \geq 0$, and $\lambda_2 \geq 0$.
`theta`	Clayton copula dependence parameter, $\theta > 0$.

Details

Stress–Strength Reliability under Clayton Copula with MWD Marginals

The stress–strength reliability is defined as $R = P(X > Y)$, which can be expressed using the joint distribution induced by the Clayton copula.

In copula form, the reliability is computed as

$$R = \int_0^\infty F_X(x)^{-(\theta+1)} \left(F_X(x)^{-\theta} + G_Y(x)^{-\theta} - 1\right)^{-\left(\frac{1}{\theta}+1\right)} f_X(x)\, dx,$$

which can also be written as

$$R = \int_0^1 t^{-(\theta+1)} \left(t^{-\theta} + G_Y(F_X^{-1}(t))^{-\theta} - 1\right)^{-\left(\frac{1}{\theta}+1\right)} dt,$$

where $F_X(x) = 1 - \exp(-a_1 x^{b_1} e^{\lambda_1 x})$ and $G_Y(y) = 1 - \exp(-a_2 y^{b_2} e^{\lambda_2 y})$.

Further theoretical details can be found in `vignette("ssr-theory", package = "SSReliabilityClaytonMWD")` and Kizilaslan (2026).

Value

A list identical to the output of `stats::integrate`:

`value`	Numerical value of the integral (reliability value).
`abs.error`	Estimated absolute error of the numerical integration.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv:2604.12130

Examples

```
a1 <- 0.75; b1 <- 1.5; lambda1 <- 0.6
a2 <- 1.2; b2 <- 0.5; lambda2 <- 0.9
theta <- 1 # 2, 3, 4, 5
R <- Reliability_Clayton_MWD(a1, b1, lambda1, a2, b2, lambda2, theta)
R$value
# approximated reliability R based on MC method
R_MC <- Reliability_Clayton_MWD_MC(a1, b1, lambda1, a2, b2, lambda2, theta, N = 50000)
R_MC$R
```

`Reliability_Clayton_MWD_MC` *Monte Carlo Estimation of Stress–Strength Reliability (MWD + Clayton Copula)*

Description

Computes an approximate value of the stress–strength reliability $R = P(X > Y)$ using Monte Carlo integration method under the modified Weibull model with dependence induced by a Clayton copula.

It calculates an approximate value of R using the Monte Carlo integration method based on a random generated MWD sample from $MWD(a_1, b_1, \lambda_1)$.

Usage

```
Reliability_Clayton_MWD_MC(a1, b1, lambda1, a2, b2, lambda2, theta, N = 10000)
```

Arguments

`a1, b1, lambda1`	Parameters of the strength variable X, with $a_1 > 0$, $b_1 \geq 0$, and $\lambda_1 \geq 0$.
`a2, b2, lambda2`	Parameters of the stress variable Y, with $a_2 > 0$, $b_2 \geq 0$, and $\lambda_2 \geq 0$.
`theta`	Clayton copula dependence parameter, $\theta > 0$.
`N`	Integer. Number of Monte Carlo samples from $X \sim MWD(a_1, b_1, \lambda_1)$.

Details

Monte Carlo Estimation of Stress–Strength Reliability under Clayton Copula

The approximate stress–strength reliability R is approximated via Monte Carlo integration:

$$R \approx \frac{1}{N}\sum_{i=1}^{N} T(x_i; \mathbf{\Omega_1}, \mathbf{\Omega_2}, \theta) = \tilde{R},$$

where $\mathbf{\Omega_1} = (a_1, b_1, \lambda_1)$ and $\mathbf{\Omega_2} = (a_2, b_2, \lambda_2)$ denote the marginal parameter vectors. The function

$$T(x; \mathbf{\Omega_1}, \mathbf{\Omega_2}, \theta) = F_X(x)^{-(\theta+1)} \left(F_X(x)^{-\theta} + G_Y(x)^{-\theta} - 1\right)^{-\left(\frac{1}{\theta}+1\right)},$$

where $x_1, \ldots, x_N$ is a random sample generated from the Modified Weibull distribution $MWD(a_1, b_1, \lambda_1)$.

The accuracy of the approximation improves as sample size N increases. In practice, values around $N = 10^5$ typically provide stable results.

Further details can be found in Kizilaslan (2026).

Value

A list containing:

`R`	Approximated reliability value based on N generated random samples.
`R_MCsample`	Monte Carlo approximate of the reliability R based on N generated samples.
`sample`	Generated random sample from $MWD(a_1, b_1, \lambda_1)$ with N size.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv:2604.12130

Examples

```
# example code
a1 <- 0.75; b1 <- 1.5; lambda1 <- 0.6
a2 <- 1.2; b2 <- 0.5; lambda2 <- 0.9
theta <- 1 # 2, 3, 4, 5
R <- Reliability_Clayton_MWD(a1, b1, lambda1, a2, b2, lambda2, theta)
R$value
# approximated reliability R based on MC method
R_MC <- Reliability_Clayton_MWD_MC(a1, b1, lambda1, a2, b2, lambda2, theta, N = 50000)
R_MC$R
```

rMweibull_Clayton *Random Generation for MWD Marginals via Clayton Copula*

Description

Generates bivariate random samples from a dependent stress–strength model where both marginals follow the Modified Weibull Distribution (MWD), and the dependence structure between the variables is modeled using a Clayton copula.

Generates bivariate random samples from a dependent stress–strength model where both marginals follow the Modified Weibull Distribution (MWD), and dependence between variables is modeled using a Clayton copula.

Usage

```
rMweibull_Clayton(n, a1, b1, lambda1, a2, b2, lambda2, theta)
```

Arguments

`n`	Integer. Number of observations to be generated.
`a1, b1, lambda1`	Parameters of the strength variable X, with $a_1 > 0$, $b_1 \geq 0$, and $\lambda_1 \geq 0$.
`a2, b2, lambda2`	Parameters of the stress variable Y, with $a_2 > 0$, $b_2 \geq 0$, and $\lambda_2 \geq 0$.
`theta`	Clayton copula dependence parameter, $\theta > 0$.

Details

Bivariate Random Data Generation under Clayton Copula with MWD Marginals

This function generates dependent uniform variables using the Clayton copula, which are then transformed via inverse CDFs of the Modified Weibull marginals to obtain (X, Y).

Further details are provided in Kizilaslan (2026).

Value

A list containing:

A list containing:

`U`	Uniform samples used in the copula construction.
`V`	Dependent uniform samples generated via the Clayton copula.
`X`	Simulated observations from $X \sim \mathrm{MWD}(a_1, b_1, \lambda_1)$ obtained by transforming U.
`Y`	Simulated observations from $Y \sim \mathrm{MWD}(a_2, b_2, \lambda_2)$ obtained by transforming V.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv:2604.12130

Examples

```
set.seed(123)
n <- 50
a1 <- 0.75; b1 <- 1.5; lambda1 <- 0.6
a2 <- 1.2; b2 <- 0.5; lambda2 <- 0.9
theta <- 1 # 2, 3, 4, 5
# data generation
dat <- rMweibull_Clayton(n, a1, b1, lambda1, a2, b2, lambda2, theta)
str(dat)
```

TerkosDam *Terkos Dam Data*

Description

Terkos Dam is one of the largest dams supplying Istanbul, Türkiye, and is located on the European side. The dataset consists of daily occupancy rates of Istanbul's dams, retrieved in March 2026 from Istanbul Metropolitan Municipality datasets website https://data.ibb.gov.tr/en.

Usage

```
TerkosDam
```

Format

A numeric vector of length 95, representing monthly average occupancy rates.

Details

The data span the period from late October 2000 to mid-February 2024. Monthly average occupancy rates are computed based on the daily data for the period September-December of each year, resulting in a total of 95 observations.

Source

Istanbul Metropolitan Municipality Open Data Portal https://data.ibb.gov.tr/en. Licensed under CC BY 4.0.

theta_Ktau_estimate *Kendall's Tau-based Estimation of the Clayton Copula Parameter*

Description

Estimates the dependence parameter θ of the Clayton copula using Kendall's tau-based moment estimator.

Usage

```
theta_Ktau_estimate(data)
```

Arguments

data	A list containing two numeric vectors: X (strength) and Y (stress).

Details

Kendall's Tau Estimator for the Clayton Copula Parameter

The estimator is derived from the relationship between Kendall's tau and the Clayton copula parameter: $\tau = \theta/(\theta + 2)$.

Value

A numeric scalar giving the estimate of θ based on Kendall's tau (τ).

Examples

```
set.seed(123)
n <- 50
a1 <- 0.75; b1 <- 1.5; lambda1 <- 0.6
a2 <- 1.2; b2 <- 0.5; lambda2 <- 0.9
theta <- 5 # 1, 2, 3, 4
# data generation
dat <- SSReliabilityClaytonMWD::rMweibull_Clayton(n, a1, b1, lambda1, a2, b2, lambda2, theta)
theta_Ktau_estimate(dat)
```

WLSE_clayton_onestep — *One-Step Weighted Least Squares Estimation of the Clayton Copula Parameter*

Description

Computes a one-step weighted least squares estimator (WLSE) of the Clayton copula dependence parameter θ. The estimator is obtained via a second-order Taylor expansion of the Clayton copula $C_\theta(u,v)$ around an initial value θ_0, typically the Kendall's tau-based moment estimate.

Usage

```
WLSE_clayton_onestep(par, x, y, estimates)
```

Arguments

`par`	Numeric scalar. Initial estimate of θ, typically obtained from Kendall's tau.
`x`	Numeric vector. Observations of the strength variable X.
`y`	Numeric vector. Observations of the stress variable Y.
`estimates`	A named list of marginal parameter estimates: (a_1, b_1, λ_1) for strength and (a_2, b_2, λ_2) for stress.

Details

One-Step WLSE Estimator for the Clayton Copula Parameter

The one-step estimator is constructed by substituting a second-order Taylor expansion of the Clayton copula $C_\theta(u,v)$ into the weighted least squares estimating equation, evaluated at θ_0, and solving analytically for θ.This avoids iterative numerical optimisation and yields a closed-form estimation of θ.

Further theoretical details are provided in Kizilaslan (2026).

Value

Numeric scalar. One-step WLSE estimate of the dependence parameter θ.

References

Kizilaslan, F. (2026). *Reliability estimation in dependent stress–strength model with Clayton copula and modified Weibull margins*. arXiv:2604.12130

See Also

`LSE_clayton_onestep` for the LSE version, `theta_Ktau_estimate` for the Kendall's Tau-based estimate.

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